Imputing the True Wind from the Apparent While Under Motor

James K. Lowden

Manhattan Sailing Club

While motoring, a sailor watches the apparent wind, looking for a chance to set sail again. The easy computation true = apparent - motor is accurate only if the apparent is directly on the nose. This article may lend some insight into how to think about the true wind while under motor.

Calculating the true wind from the apparent

An Island Packet 380 motoring North at 4 knots SOG observes 5 knots of apparent wind off the starboard bow. What is the true wind?

The apparent wind can be decomposed into two vectors: the wind created by the motor pushing the yacht forward at 4 knots, and the true wind. The equation is

$$V_{apparent} = V_{motor} + V_{true}$$

Solving for V_{true}

 $V_{true} = V_{apparent} - V_{motor}$

Vectors can be added (and subtracted) geometrically. Draw a vector for the North wind owing to the motor, and another representing the apparent wind. The distance between the origins of the two is the true wind.



One step beyond geometry is trigonometry, with our friends sine and cosine. The three key ratios are

$$sine = \frac{opposite}{hypotenuse}$$
 $cosine = \frac{adjacent}{hypotenuse}$ $tangent = \frac{opposite}{adjacent}$

in terms of our motoring yacht



On the boat, one observes speed over ground, wind speed, and wind direction. Keeping with our 3-4-5 example, we're travelling at 4 knots and have 5 knots apparent NNE. The exact wind angle is 37°. The true wind is found using the sine

 $true = \sin(37) \cdot 5 = 0.6 \cdot 5 = 3$

Of course the wind won't always arrive from the East in nice, round numbers. What if we see 12 knots at 60° ?



A roundabout way, using only the sine

To solve for the true wind using only the sine, we need *right* triangles (There is a fancier formula, shown later.) We can make two right triangles by dividing the one we have:



Because we've defined the divider as perpendicular to the apparent wind, we have two right triangles. For the smaller one, with the motor-induced wind is the hypotenuse and the divider is the opposite leg. We can compute the length of the divider using the sine

$$divider = \sin(60) \cdot 4 = 0.866 \cdot 4 = 3.464$$

With that information, we can walk around the two triangles computing lengths using the Pythagorean Theorem, $c^2 = a^2 + b^2$. Part of the apparent wind is in the little triangle.

 $4^2 = 3.464^2 + little apparent^2$

solving for the apparent

little apparent = $\sqrt{4^2 - 3.464^2} = \sqrt{16 - 12} = \sqrt{4} = 2$ knots

The large triangle's apparent wind segment is therefore 10 = 12 - 2. The true wind is its hypotenuse.

$$true = \sqrt{10^2 + 3.464^2} = \sqrt{100 + 12} = \sqrt{112} = 10.58$$
 knots

Now we're almost home. What of the *angle* of the true wind? Back to trigonometry. Each of the basic trig ratios — sine, cosine, and tangent — has an inverse that yields the angle for the ratio. We solve for the

angle between the true and apparent winds by computing the sine and applying its inverse, the arcsine¹

$$angle = \arcsin\left(\frac{3.464}{10.58}\right) = 19^{\circ}$$

so the true wind is

true wind angle = $60 + 19 = 79^{\circ}$

Let's take another look.



Only 1.4 knots of the motor's speed contributes to the observed apparent wind. The naïve calculation 8 = 12 - 4 leads one to conclude there's not enough wind to sail in. The correct one makes it clear there is.

The direct way: Law of Cosines

The magnitudes of the motor and apparent wind are known, as is the angle between them. Any trig student will recognize this as an opportunity to apply the *Law of Cosines*. The formula is:

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

Plugging in our data

$$true = \sqrt{4^2 + 12^2 - 2 \cdot 4 \cdot 12 \cdot \cos(60)}$$
$$true = \sqrt{16 + 144 - 96 \cdot 0.5} = 10.58 \text{ knots}$$

Now that we know the lengths of all three sides, we can also compute the angle of the true wind. The ratio of the sine of an angle to its opposite leg is constant

 $\frac{\sin(M)}{motor} = \frac{\sin(T)}{true}$ which is to say $\frac{\sin(M)}{4} = \frac{\sin(60)}{10.58}$ $\sin(M) = 4 \cdot \frac{\sin(60)}{10.58} = \frac{4 \cdot 0.866}{10.58} = 0.327$ $M = \arcsin(0.327) = 19^{\circ}$

and again the true wind is

true wind angle = $60 + 19 = 79^{\circ}$

The length of the arc on such a circle is the arcsine.

¹ Why are inverse functions are called "arc"? Imagine a *unit circle* with an angle at its center

Calculating the apparent wind from the true at different angles

The figure below shows apparent wind for different angles of true wind at 10-degree intervals. The motor continues to be 4 knots, and true wind 10 knots. The blue, green, and red lines illustrate the true wind at 10° , 90° , and 170° respectively.



The magnitudes of the motor and true wind are constant, and the angle between them is known. The formula again:

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

or, as it applies here

$$apparent^{2} = motor^{2} + true^{2} - 2 \cdot motor \cdot true \cdot \cos(A)$$

where A is the angle opposite *apparent*, the one between the two known vectors, *motor* and *true*. For example, to solve for the the apparent wind velocity with true wind at 10°

 $apparent^{2} = 4^{2} + 10^{2} - 2 \cdot 4 \cdot 10 \cdot \cos(180 - 10)$

 $apparent^2 = 116 - 80 \cdot \cos(170)$

apparent = $\sqrt{116 - 80 \cdot -0.9848} = \sqrt{116 - -78.7846} = \sqrt{194.78} = 13.96$ knots

which is to say that 10° off the bow is pretty much the same as head-on in this wind with this motor.

True°	Apparent°	Apparent (kt.)
0	0	14.00
10	7	13.96
20	14	13.83
30	22	13.61
40	29	13.31
50	36	12.94
60	44	12.49
70	52	11.97
80	60	11.40
90	68	10.77
100	77	10.10
110	94	9.41
120	97	8.72
130	108	8.04
140	120	7.40
150	133	6.84
160	148	6.39
170	163	6.10
180	180	6.00

The table below shows the angle and speed of the apparent wind for every 10° as described above (motor 4 kt, true wind 10 kt).

As the apparent wind swings abeam, the motor's contribution becomes negligible. At 50° , the motor supplies 3 knots of apparent wind, at 70° 2 knots, at 85° 1 knot, and at 100° zero.

The rough-and-ready conclusion: if while motoring you see enough wind to sail in at an angle you could trim for, you can turn the motor off. Wind abeam is really there, regardless of RPMs.